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# One-dimensional dynamics of domain walls in two-layer ferromagnet structure with different parameters of magnetic anisotropy and exchange

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ABSTRACT

One-dimensional non-linear dynamics of domain walls (DW) under the influence of external constant magnetic field in a two-layer ferromagnet with different values of magnetic anisotropy and exchange parameters in the layers is theoretically studied in the article. Using analytical methods a motion equation for the DW centre coordinate, its stationary velocity after transition from one layer to another and its minimum velocity necessary for DW transition from one layer to another are found. It is shown that for the case of small defects, the results obtained analytically are well coordinated with the numerical ones.



## 1. Introduction

Photon and magnonic crystals have been widely researched recently. They may be related to multilayer film structures (one-dimensional superstructures) which are in fact alternating layers of two materials with different physical properties. One-dimensional models (sinusoidal and rectangular profile superlattices) are used to describe the dynamics of linear and non-linear magnetization waves in such layered structures [1–4]. For example, a rectangular profile superlattice model describes a layered structure with alternation of two different layers with one or several magnetic parameters, whereas the transition from layer to layer occurs at a distance of the atomic order. In the continuum model magnetic parameters of such a ferromagnetic superlattice can be piecewise described by a constant function which obtains a zero width of the transition area. It should be noted that a local spatial modulation of magnetic material parameters can be obtained in studying two- and three-layer magnetic structures (e.g. [5–9]). A local spatial modulation of magnetic parameters of material can be also received as applied in external (mechanical, thermal or light) actions [10,11].

The influence of local and periodic one-dimensional spatial modulation of magnetic parameters on the distribution pattern, spectrum and damping of spin waves is well investigated [12,13]. Studying the one-dimensional dynamics of DW leads to searching the solution for the modified sine-Gordon (SG) equation type with a variable coefficient which is of great importance for modern physics [14–16]. In a weakly inhomogeneous case it may be considered that the presence of perturbations does not considerably change the form of DW mainly affecting their dynamics. In a strongly inhomogeneous case the form of DW must be changed greatly and the excitation modes within their boundaries and radiation of bulk spin waves should be expected. Due to the task set, the authors considered the modulation of separate parameters of the magnetic system. For example, the modulation of the magnetic anisotropy for two and three-layer magnet was taken into account and studied both by analytical [5,8,9,17] and numerical [6,18,19] methods. For a three-layer magnet the modulation and an exchange parameter were also considered [7,8,20,21]. In the article the influence of spatial modulation parameters of magnetic anisotropy and exchange on the dynamics of DW in a two-layer ferromagnet are investigated.

## 2. Formulation of the problem: equations of motion

Let us consider an infinite ferromagnet whose crystallographic axes ($a$,$b$,$c$) coincide with the Cartesian coordinate axes ($x$, $y$, $z$). Taking into account the exchange interaction, anisotropy, Zeeman energy and damping in the magnet energy density, we can write the motion equation for magnetization in angular variables of a ferromagnetic vector $m = m(0, \cos\theta, \sin\theta)$ in the dimensionless form [20]

$$\frac{\partial}{\partial x}\left(g(x)\cdot\frac{\partial\theta}{\partial x}\right)-\frac{\partial^2\theta}{\partial t^2}-\frac{1}{2}f(x)\sin 2\theta = h\sin\theta+\alpha\frac{\partial\theta}{\partial t} \quad (1)$$

where $\theta$ is an angle in the $yz$ plane between the direction of the magnetic moment vector $m$ and the axis of easy magnetization

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($Oz$ axis), functions $f(x)$ and $g(x)$ determine spatial modulation of magnet anisotropy and exchange constants, respectively; $h$ is the external magnetic field; $\alpha$ is the damping constant. The coordinate $x$ is normalized to the quantity $\delta_0$, where $\delta_0$ is the width of a static Bloch DW, the time $t$ is normalized to $\delta_0/c$, where $c$ is the limiting Walker velocity of stationary motion of the DW [22]. Let us consider the case of two-layer ferromagnet when the functions $f(x)$ and $g(x)$ are described in a step form [6,17]

$$g(x)=\begin{cases}1+\eta, & x\geq 0,\\ 1, & x<0,\end{cases} \quad f(x)=\begin{cases}1+\varepsilon, & x\geq 0\\ 1, & x<0\end{cases} \tag{2}$$

It should be noted that the equation of the (1) type may be obtained for weak ferromagnets and ferrites as well, where $c$ is a spin-wave velocity. Eq. (1) which is thoroughly studied today is a modified SG equation with variable coefficients [14,15]. Although there is a well-developed perturbation theory for this equation [15,17,23,24], numerical methods are to be used for the given case of arbitrary values of $\varepsilon$ and $\eta$ parameters [18,20,25,26]. The parameter of the system $(1+\eta)$ is always above zero. The cases $\eta=\alpha=h=0$ and $|\varepsilon|\ll 1$ were considered earlier with the help of the perturbation theory for solitons [17] whereas the case where the parameter $\varepsilon$ takes arbitrary values was solved numerically in [18].

Let the Bloch DW ($\pi$-kink) be at the initial moment of time,

$$\theta_k(x,0)=2\arctan(e^{(x-q)}) \tag{3}$$

where $q$ is a coordinate of the DW centre. We search for the analytical solving of Eq. (1) in $\alpha=h=0$ in the following form: $\theta=\theta_s(x-q)$. This function satisfies the boundary conditions that $\theta_s(x\to-\infty)=0,\ \theta_s(x\to\infty)=\pi,\ \theta_s'(x\to\pm\infty)=0$. Using the perturbation theory for solitons [20] the motion equation for the DW centre coordinate is searched as follows:

$$m^*\frac{d^2q}{dt^2}=-2\int_{-\infty}^{\infty}F(x)\frac{\partial\theta}{\partial x}\,dx \tag{4}$$

where $m^*=\int_{-\infty}^{\infty}(\partial\theta_s/\partial x)^2\,dx$ is the effective DW mass, with $m^*=8$,

$$F(x)=-\frac{1}{2}\varepsilon H(x)\sin 2\theta+\eta H(x)\frac{\partial^2\theta}{\partial x^2}+\eta H'(x)\frac{\partial\theta}{\partial x} \tag{5}$$

$H(x)$—Heaviside step function:

$$H(x)=\begin{cases}0, & x<0\\ 1, & x\geq 0\end{cases} \tag{6}$$

As a result we obtain the motion equation as

$$m^*\frac{d^2q}{dt^2}=-(2\varepsilon+2\eta)\,\mathrm{sech}^2(q) \tag{7}$$

Eq. (7) allows to receive the dependences of velocity and DW centre coordinates from time for different values of $\varepsilon$ and $\eta$ parameters. At $\varepsilon+\eta\geq 0$ elastic reflection from the area of the local spatial modulation of magnetic parameters is observed whereas at $\varepsilon+\eta\leq 0$ further velocity increase occurs.

When DW moving by inertia the law of energy conservation is similarly [17] used for finding its steady velocity after transition through the layer boundaries

$$\frac{M_0c_0^2}{(1-\upsilon_0^2/c_0^2)^{1/2}}=\frac{Mc^2}{(1-\upsilon^2/c^2)^{1/2}} \tag{8}$$

where $M_0$ and $M$ are the effective DW masses, $c_0$ and $c$ are the limiting velocities in the first and second layers of magnet. In our case $M/M_0=(1+\varepsilon)^{1/2}/(1+\eta)^{1/2}$ and $c/c_0=(1+\eta)^{1/2}$. After simple transformation we have

$$\upsilon^2=(1+\eta)\cdot[1-(1+\eta)(1+\varepsilon)(1-\upsilon_0^2)] \tag{9}$$

Note that the formula (9) at $\eta=0$ is transformed to the known formula [17].

In case $\varepsilon+\eta>0$ there exists a minimum DW velocity $\upsilon_{\min}$ necessary for passing through the boundary between the layers. Physically it is explained by the fact that in overcoming the barrier a part of the DW kinetic energy is spent on increasing its potential energy due to value changes in the system parameters. The minimum velocity value can be obtained from Eq. (9) by equating the DW motion velocity to zero after passing through the boundary between the layers

$$\upsilon_{\min}=\{(\varepsilon+\eta+\varepsilon\eta)/[(1+\eta)(1+\varepsilon)]\}^{1/2} \tag{10}$$

Note that the parameters $\varepsilon$ and $\eta$ amount the same in the $\upsilon_{\min}$ value.

## 3. Result of numerical calculations

Eq. (1) was solved numerically as well. The explicit scheme of Eq. (1) integration was used [20]. At the starting point of time the DW of the type (3) is set far from the boundary (or defect) between the layers and moves with the initial speed $\upsilon_0$. Discretization of the equation was carried out according to the standard five-point scheme of the "cross" type. For our calculations we use a uniform grid with a step $\xi$ in the coordinate $x$: $\{x_i=\xi\cdot i,\ i=0,\pm1,\ldots,\pm N_x\}$, and with a step $\tau$ in the time $t$: $\{t_n=\tau n,\ n=0,1,\ldots,N_t\}$, where $N_x$ and $N_t$ are the numbers of grid points. By satisfying the convergence condition of the explicit scheme $\tau/\xi\leq 0.25$, we calculated the angle $\theta$ at the next moment of time.

First let us consider the dynamics of DW passing by inertia through the boundary between the layers. At the moment of DW passing the boundary between the layers, low-amplitude waves extending to the right and left of it appear, the boundary velocity drops or rises and goes to a new stationary value. For the case of low velocities and low values of $\eta$ and $\varepsilon$ parameters, the results agree with each other rather well (Fig. 1).

In Fig. 2 the numerically obtained dependences of the stationary DW velocity from the $\eta$ and $\varepsilon$ parameters are shown. The results received are similar to the ones obtained using the formula (9). Some difference is explained by the fact that in calculation the changes in the DW structure are taken into account, thus, the velocity values are less than the analytical ones. In Fig. 3 the dependence of the minimum velocity necessary for DW passing the local spatial modulation of magnetic material parameters from the $\eta$ and $\varepsilon$ parameters is given. At low $\eta$ and $\varepsilon$ values the numerically obtained results coincide with the analytical values of the (10) formula. At high $\eta$ and $\varepsilon$ parameters the numerical $\upsilon_{\min}$ value is considerably less than the analytical one. It can be attributed by more accurate numerical account of changes in the DW structure. The difference also lies in the fact that in the numerical experiment the parameter $\varepsilon$ influences the $\upsilon_{\min}$ value more than the parameter $\eta$.

The DW dynamics is considered under the influence of the external constant magnetic field. It is known that in this case the stationary DW velocity may be found by the formula [20]

$$\upsilon=\chi/(1+\chi^2)^{1/2} \tag{11}$$

where $\chi=h/\alpha$. While passing through the local spatial modulation of magnetic material parameters the DW velocity after a long period of time is to be quite constant and is equal to

$$\upsilon^*=\upsilon_0\left[\frac{(1+\eta)\cdot(1+h^2)}{1+\varepsilon+h^2}\right]^{1/2} \tag{12}$$

From the formula (12) we obtain a simple dependence in case $\varepsilon=0$: $\upsilon^*=\upsilon_0\cdot(1+\eta)^{1/2}$. In Fig. 4 the dependences $\upsilon^*$ from the $\eta$

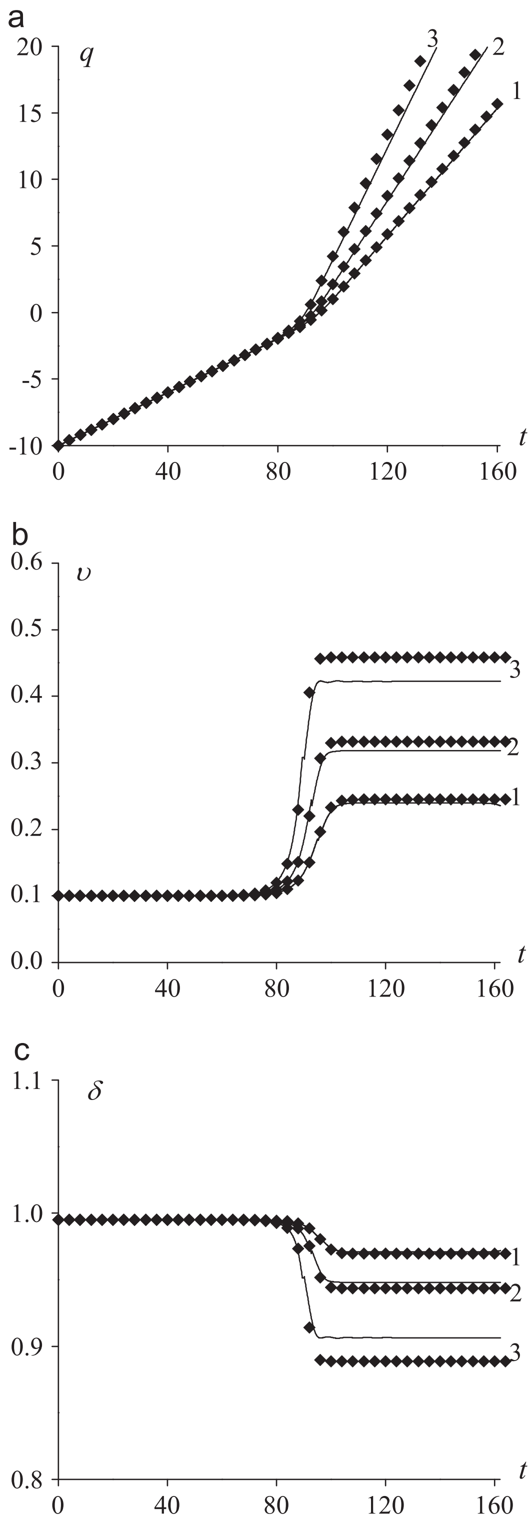


**Fig. 1.** (a) Dependence of the DW centre, (b) velocity and (c) width from time in case $\upsilon_0 = 0.1$ (1—$\varepsilon+\eta=-0.05$, 2—$\varepsilon+\eta=-0.1$, 3—$\varepsilon+\eta=-0.2$). Lines are Eq. (1) solutions, dots are Eq. (7) solutions.

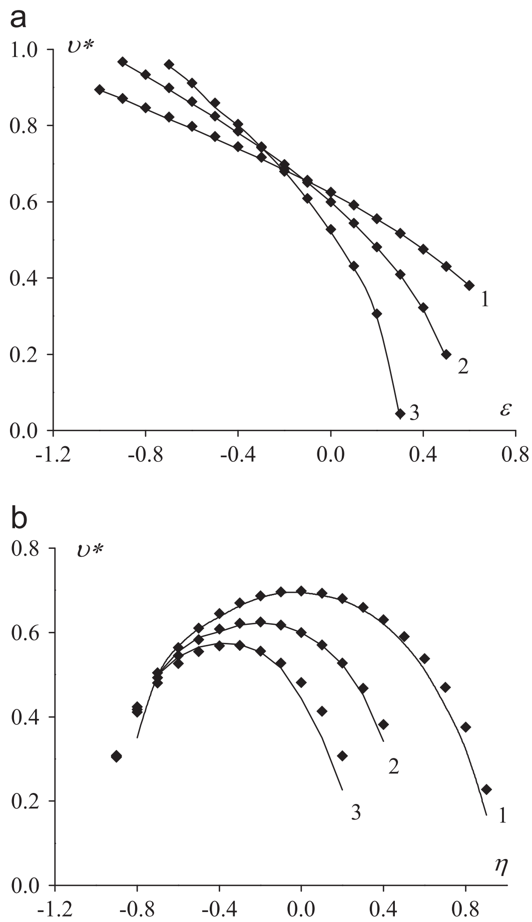


**Fig. 2.** Dependence of the stationary DW velocity from $\varepsilon$ (a) and $\eta$ (b) parameters. $\upsilon_0 = 0.6$. Lines are values obtained by the formula (9), dots are results of numerical calculations (a: 1—$\eta=-0.2$, 2—$\eta=0$, 3—$\eta=0.2$; b: 1—$\varepsilon=-0.2$, 2—$\varepsilon=0$, 3—$\varepsilon=0.2$).

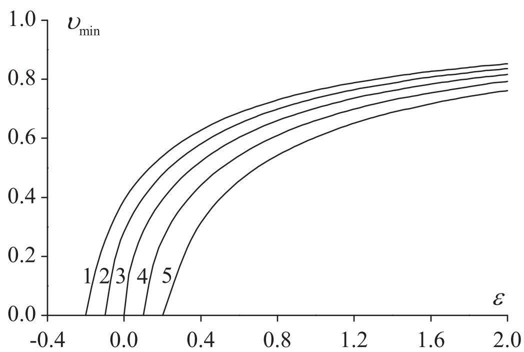


**Fig. 3.** Dependence of minimum DW velocity $\upsilon_{\min}$, necessary for passage the transition area from parameter $\varepsilon$ at $\alpha=h=0$ (1—$\eta=-0.2$, 2—$\eta=-0.1$, 3—$\eta=0$, 4—$\eta=0.1$, 5—$\eta=0.2$).

and $\varepsilon$ parameters are shown. The results of numerical calculations and values of velocities determined by the formula (12) well conform with each other. In Fig. 5 the comparison of minimum velocities necessary for passing through the boundary between the layers for the inertia cases and in the presence of the external force is carried out. In both cases the results differ inconsiderably which point out to a weak dependence from the external force and damping.

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In case $\upsilon < \upsilon_{\min}$, $\varepsilon+\eta > 0$ in DW moving by inertia its elastic reflection of the step takes place which results in changes of its direction to the opposite one. In $h \neq 0$, $\alpha \neq 0$ the case of damping oscillations in the transition area (or pinning of DW) is possible as well (Fig. 6, see movie in Fig. 7). Here the radiation of low-amplitude waves is observed either. The dependence of the coordinate and velocity of the center of DW from time is

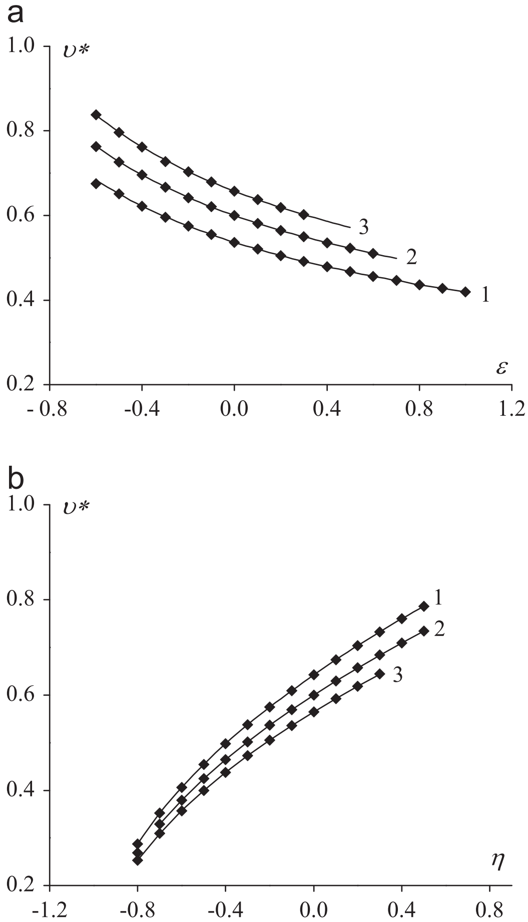


**Fig. 4.** Dependence of the limiting DW velocity from the $\varepsilon$ (a) and $\eta$ (b) parameters, $h=0.015$. Lines are values obtained by the formula (12), dots are results of numerical calculations (a: 1—$\eta=-0.2$, 2—$\eta=0$, 3—$\eta=0.2$; b: 1—$\varepsilon=-0.2$, 2—$\varepsilon=0$, 3—$\varepsilon=0.2$).

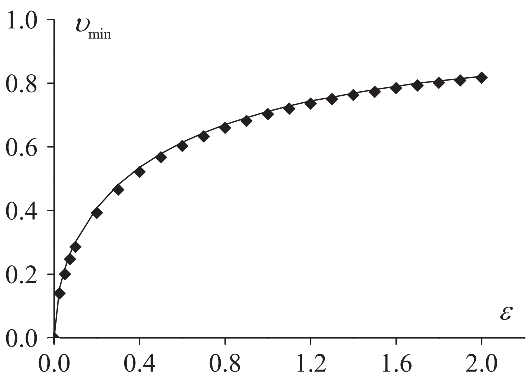


**Fig. 5.** Dependence of the minimum DW velocity $\upsilon_{\min}$, necessary for passage the transition area from the parameter $\varepsilon$ at $\eta=0$; for inertia cases a line is given ($\alpha=h=0$) and for external force $h$=0.015 dots are used.

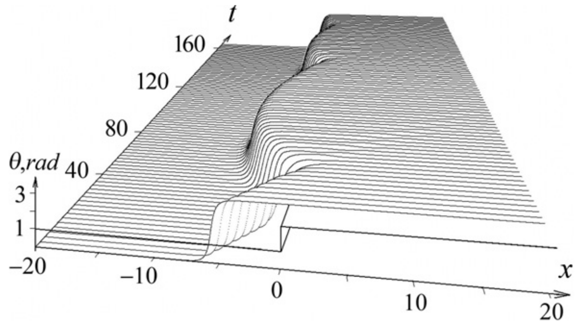


**Fig. 6.** Dynamics of DW pinning in the transition area (defect) at $\eta=1$, $\varepsilon=0$, $h=0.015$ and $\upsilon_0=0.6$.

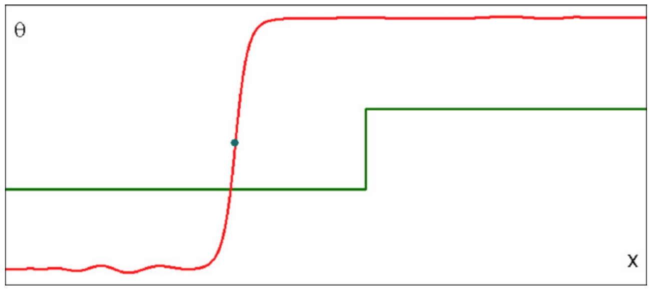


**Fig. 7.** Movie of dynamics of DW pinning in the transition area (defect) at $\eta=1$, $\varepsilon=0$, $h=0.015$ and $\upsilon_0=0.6$.

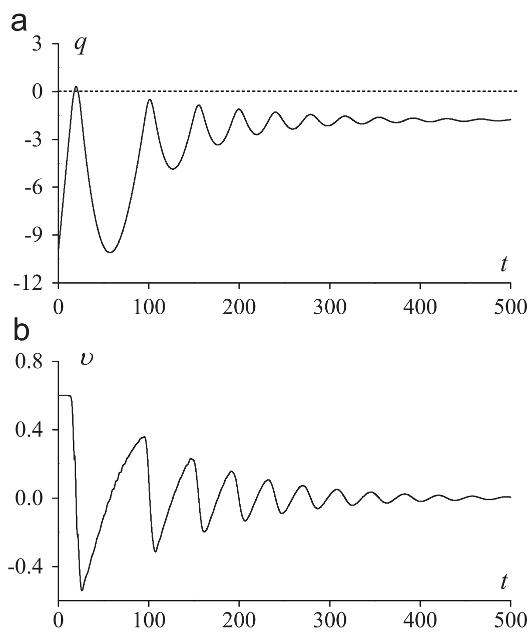


**Fig. 8.** Dependence of the coordinate $q$ (a) and velocity $\upsilon$ (b) of the DW centre from time for the case in Fig. 6. The dashed line on the $q(t)$ function is the boundary of the transition area (defect).

presented in Fig. 8(a) and (b) for the case in Fig. 6. From the figures we may conclude that the DW reaches the boundary of the transition area only at the initial time. Then we obtain the case of the asymmetric curve when the DW moves to the left under the potential force arising from the presence of inhomogeneous potential energy in the àrea of the local spatial modulation of magnetic material parameters and to the right due to the external magnetic field. Further oscillations occur near this area and may be considered harmonic after certain time ($t\sim200$). The dependence of the DW width from time may be analytically calculated by the formula [20]

$$\delta=\sqrt{1-\upsilon^2} \tag{13}$$

The numerical calculations show that the values of the DW width in the steady motion to the step area coincide numerically and analytically, however certain changes are observed in complex oscillations of the DW width after passage the boundary of this area. It is connected with the changes either in the velocity or the DW structure. The presence of DW width oscillations after passing through the boundary between the layers points to exciting internal

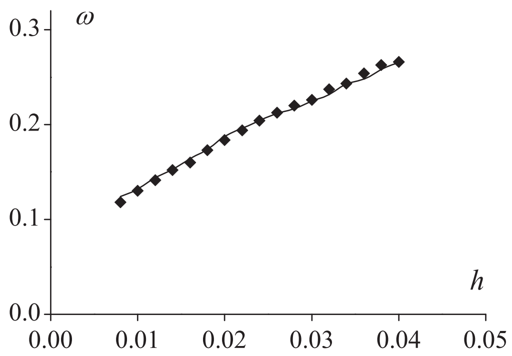


**Fig. 9.** Dependence of the DW oscillation frequency $\omega$ from the field $h$. $\eta = 4$. Lines are a transition mode, dots are a pulsation mode.

translation modes related to pulsation ones. In Fig. 9 the dependence of frequency translation and pulsation modes of DW on the parameter $h$, calculated from the dependence of $x(t)$ and $\delta(t)$ is shown. As seen from the figure, $\omega_T$ and $\omega_p$ coincide whereas the dependence $\omega$ from $h$ is similar to the form $\omega \sim h^{1/2}$.

After the stop the DW structure is greatly changed as compared to the homogeneous case (3).

## 4. Conclusion

The DW dynamics is studied in the two-layer ferromagnet with different parameters of magnet anisotropy and exchange. The dependences of the minimum DW velocities necessary for one to another layer passage from the material parameters, namely, anisotropy and exchange are obtained. Using the perturbation theory for solitons the equation for the DW centre velocity is obtained. The dependences of steady DW velocities from the material parameters in crossing from one layer to another are determined.

It is shown that the received analytical results coincide with the numerical ones for the case of small inhomogeneities in the material parameters and DW velocities. The DW pinning on the boundary between the layers is studied as well. The frequencies of DW inner oscillations modes (translation and pulsation) are determined.

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version, at http://dx.doi.org.10.1016/j.jmmm.2013.02.042.

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