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One-dimensional dynamics of magnetic inhomogeneities in a three- and five-layer ferromagnetic structure with different values of the magnetic parameters

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Abstract. The article presents the research results of localized magnetization waves dynamics in five-layer and three-layer ferromagnetic structures. Structures consisting of three or two wide identical magnetic layers separated by two or one thin magnetic layer with the magnetic parameters values changed relative to wide layers are considered. The magnetic anisotropy, exchange, and damping parameters are considered to be the functions of the coordinate perpendicular to the interfaces. Intermediate layers are assumed to be infinitely thin. The considered magnetic material has homogeneous magnetic parameters everywhere, except for the planes corresponding to the interlayers. Using the approximate collective-coordinate method, the dynamics of coupled nonlinear magnetization waves localized on thin layers is theoretically studied.

1. Introduction

Multilayer magnetic structures [1] often represent periodically alternating layers of two materials with different physical properties. With regard to the possibility of their practical application, we study the dynamics of spin waves and magnetic inhomogeneities propagating in such systems both along and perpendicular to the interfaces of the layers. In the second case, one-dimensional models are often used [2, 3]. The study of one-dimensional models allows us to understand the effect of different magnetic parameters on the process under consideration [2–5]. There are two approaches when researching the dynamics of linear and nonlinear magnetization waves propagating perpendicular to the layers. In the first of them, to describe the magnetization dynamics in a layer, the Landau-Lifshitz equation with constant material parameters is considered, and certain boundary conditions are required at the interface of the layers [6]. In the second approach spatial modulation of the material magnetic parameters values is introduced [7, 8]. The influence of local and periodic one-dimensional spatial modulation of the material magnetic parameters on the propagation nature, spectrum and damping of spin waves, and on high-frequency properties has been studied quite well [2]. In such systems it is possible to generate localized magnetization waves (LMW) of the magnetic soliton and breather types in the magnetic defect region [7–10]. Special interest in magnetic solitons and breathers is currently associated with the appearance of new experimental techniques that allow to study formation and propagation of localized magnetization waves of nanometer dimensions and such waves interaction with domain walls (DW) [11–16].

Under certain conditions, the solution of this problem leads to the solution of a sine-Gordon equation with variable coefficients. This equation is of interest for many areas of modern physics [17]. For simplicity, as a rule there was considered a spatial modulation of only some magnetic system parameters. The modulation of magnetic anisotropy was often taken into account for the case of a two, three-, and five-layer magnetic. These problems were studied both by analytical and numerical methods [7, 18]. This paper presents a theoretical study of the dynamics of localized magnetic inhomogeneities of multisoliton type in five-layer and three-layer ferromagnetic structures. The magnetic anisotropy, exchange, and damping parameters are considered to be the functions of the coordinate perpendicular to the interfaces.

2. Equations, results and discussion

Let us consider a five-layer ferromagnetic structure. This structure consists of three wide identical magnetic layers separated by two thin magnetic layers with altered values of anisotropy, exchange, and damping parameters. These parameters are considered to be the functions of the coordinate x perpendicular to the interfaces. Intermediate layers are assumed to be infinitely thin. The considered magnetic material has homogeneous magnetic parameters everywhere, except for the planes corresponding to the interlayers. We will continue by studying the dynamics of localized magnetization waves located in the yz plane. Usually, when solving dynamic problems, it is convenient to go to the spherical coordinates of the magnetization vector **M** ($\sin\varphi$, $\cos\varphi\sin\theta$, $\cos\varphi\cos\theta$). Here $0 \leq \theta \leq 2\pi$ is the angle in the yz plane between the magnetic moment vector direction and the axis of easy magnetization (axis Oz), $-\pi/2 < \varphi < \pi/2$ is the angle describing the exit of **M** from the domain wall plane. Considering the exchange interaction, anisotropy and assuming $\varphi << 1$ in the magnetic energy density, the motion equation for the magnetization in the angular variables in the presence of two infinitely thin magnetic layers can be represented in the following dimensionless form:

$$u_{tt} - \{[1 + \gamma_1\delta(x) + \gamma_2\delta(x-d)]u_x\}_x + [1 - \varepsilon_1\delta(x) - \varepsilon_2\delta(x-d)]\sin u \\ = -2h\sin(u/2) - \alpha[1 + \beta_1\delta(x) + \beta_2\delta(x-d)]u_t, \tag{1}$$

where $u = 2\theta(x,t)$, $\delta(x)$ is the Dirac delta function, ε_1 and ε_2, γ_1 and γ_2 are the spatial modulation amplitudes of the anisotropy and exchange constants on the first and second thin layer, d is the distance between thin layers, h – normalized magnitude of the external magnetic field. Inhomogeneous dissipation is written as $\alpha[1 + \beta_1\delta(x) + \beta_2\delta(x - d)]$, where α – the dissipation coefficient in the thick layer, β_1 and β_2 take into account the change in dissipation in the thin layers. Equation (1) is a modified sine-Gordon equation. The equation of the form (1) can also be obtained for the case of two-sublattice ferrimagnets and weak ferromagnets.

Let us apply the approximate collective-coordinate method used earlier to analyze the oscillations of nonlinear LMW on identical infinitely thin magnetic layers [18]. We will take into account the LMW presence (or impurity modes) by introducing two collective variables $a_1 = a_1(t)$ and $a_2 = a_2(t)$, which are the amplitudes of these waves. We take the impurity modes in the form of [9, 18]:

$$u_a = u_1 + u_2 = a_1\exp(-\varepsilon_1|x|/2) + a_2\exp(-\varepsilon_2|x-d|/2). \tag{2}$$

The magnitude of the impurity mode should greatly decrease to the neighboring impurity. In the framework of the considered approximation $\varepsilon_{1,2}$, $|a_{1,2}|$, $|h|$, $\alpha << 1$. Then the nonlinear term in Lagrangian, leading to equation (1), can be expanded into a Taylor series to second-order terms in ε. Substituting (2) into Lagrangian and Rayleigh dissipative function, corresponding to equation (1), after integration leads to a new effective Lagrangian and Rayleigh function, which are already functions of the new variables a_1 and a_2. The motion equations for a_1 and a_2 are obtained by substituting the effective Lagrangian and Rayleigh function into the Lagrange equations of the second type. They have the following form:

$$\ddot{a}_1 + \alpha\dot{a}_1 + a_1\omega_1^2 + a_2k_1 + \alpha[\dot{a}_1(\beta_{11} - \beta_{12}\varepsilon_2 E) + \dot{a}_2(\beta_{12} - \beta_{22}\varepsilon_2 E)]\varepsilon_1 = 0, \quad (3)$$

$$\ddot{a}_2 + \alpha\dot{a}_2 + a_2\omega_2^2 + a_1k_2 + \alpha[\dot{a}_2(\beta_{22} - \beta_{12}\varepsilon_1 E) + \dot{a}_1(\beta_{12} - \beta_{11}\varepsilon_1 E)]\varepsilon_2 = 0, \quad (4)$$

where

$$e_n = \exp(-\varepsilon_n d/2), \qquad E = \frac{e_1 + e_2}{\varepsilon_2 + \varepsilon_1} + \frac{e_1 - e_2}{\varepsilon_2 - \varepsilon_1}, \quad (5)$$

$$\omega_1^2 = 1 \pm h - \frac{\varepsilon_1^2}{4} + \frac{[\varepsilon_2^2 E - (\varepsilon_2 - \gamma_2\varepsilon_1^2/4)e_1]\varepsilon_1 e_1}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \quad (6)$$

$$\omega_2^2 = 1 \pm h - \frac{\varepsilon_2^2}{4} + \frac{[\varepsilon_1^2 E - (\varepsilon_1 - \gamma_1\varepsilon_2^2/4)e_2]\varepsilon_2 e_2}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \quad (7)$$

$$k_1 = \frac{[(\varepsilon_1 - \gamma_1\varepsilon_2^2/4)\varepsilon_2 E e_2 - \varepsilon_1]\varepsilon_1 e_2}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \qquad k_2 = \frac{[(\varepsilon_2 - \gamma_2\varepsilon_1^2/4)\varepsilon_1 E e_1 - \varepsilon_2]\varepsilon_2 e_1}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \quad (8)$$

$$\beta_{11} = \frac{\beta_1 + \beta_2 e_1^2}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \qquad \beta_{22} = \frac{\beta_1 e_2^2 + \beta_2}{2(1 - \varepsilon_1\varepsilon_2 E^2)}, \qquad \beta_{12} = \frac{\beta_1 e_2 + \beta_2 e_1}{2(1 - \varepsilon_1\varepsilon_2 E^2)}. \quad (9)$$

When ε_2, γ_2, β_2, $a_2 = 0$ or $d \to \infty$ can be obtained from (3), (4) is an equation for describing the LMW dynamics in a three-layer ferromagnetic structure. Dynamic equations (3) and (4) are the equations of an oscillatory system with two coupled oscillators. The solution of this system in the absence of damping ($\alpha = 0$) has the form:

$$a_1 = a_{01}\cos(\Omega_1 t + \theta_1) + \eta_2 a_{02}\cos(\Omega_2 t + \theta_2), \quad (10)$$

$$a_2 = \eta_1 a_{01}\cos(\Omega_1 t + \theta_1) + a_{02}\cos(\Omega_2 t + \theta_2). \quad (11)$$

From (10) and (11) it follows that the LMW oscillations on two thin layers in the absence of damping are the sum of two harmonic oscillations. Oscillations frequencies:

$$\Omega_{1,2}^2 = \left[\omega_1^2 + \omega_2^2 \mp \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k_1k_2}\right]/2, \quad (12)$$

initial phases θ_1 and θ_2, amplitudes $|a_{01}|$ and $|\eta_2 a_{02}|$ on the first thin layer and $|\eta_1 a_{01}|$ and $|a_{02}|$ on the second, where the coefficients, determining the effect of one LMW on another, are equal:

$$\eta_1 = \frac{\omega_2^2 - \omega_1^2 - \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k_1k_2}}{2k_1}, \qquad \eta_2 = \frac{\omega_1^2 - \omega_2^2 + \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k_1k_2}}{2k_2}. \quad (13)$$

We turn to the normal or basic coordinates, each of which varies with the same frequency:

$$(a_1 - a_2\eta_2)/(1 - \eta_1\eta_2) = a_{01}\cos(\Omega_1 t + \theta_1) = \varphi_1, \quad (14)$$

$$(a_2 - a_1\eta_1)/(1 - \eta_1\eta_2) = a_{02}\cos(\Omega_2 t + \theta_2) = \varphi_2, \quad (15)$$

and dynamic equations can be reduced to two uncoupled differential equations regarding the main coordinates:

$$\ddot{\varphi}_1 + \varphi_1\Omega_1^2 = 0, \qquad \ddot{\varphi}_2 + \varphi_2\Omega_2^2 = 0. \quad (16)$$

Oscillations (10)–(11) can also be represented as beats with a beating frequency equal to the difference between the main oscillations frequencies and with an amplitude varying from $|a_{01} + \eta_2 a_{02}|$ to $|a_{01} - \eta_2 a_{02}|$ and from $|a_{02} + \eta_1 a_{01}|$ to $|a_{02} - \eta_1 a_{01}|$.

Figure 1 shows the dependence of the main oscillations frequencies on d. When ε_2, $\gamma_2 = 0$:

$$\Omega_{1,2}^2 = 1 \pm h - \varepsilon_1^2/4, 1 \pm h, \qquad \eta_1 = 0, \eta_2 = -2. \quad (17)$$

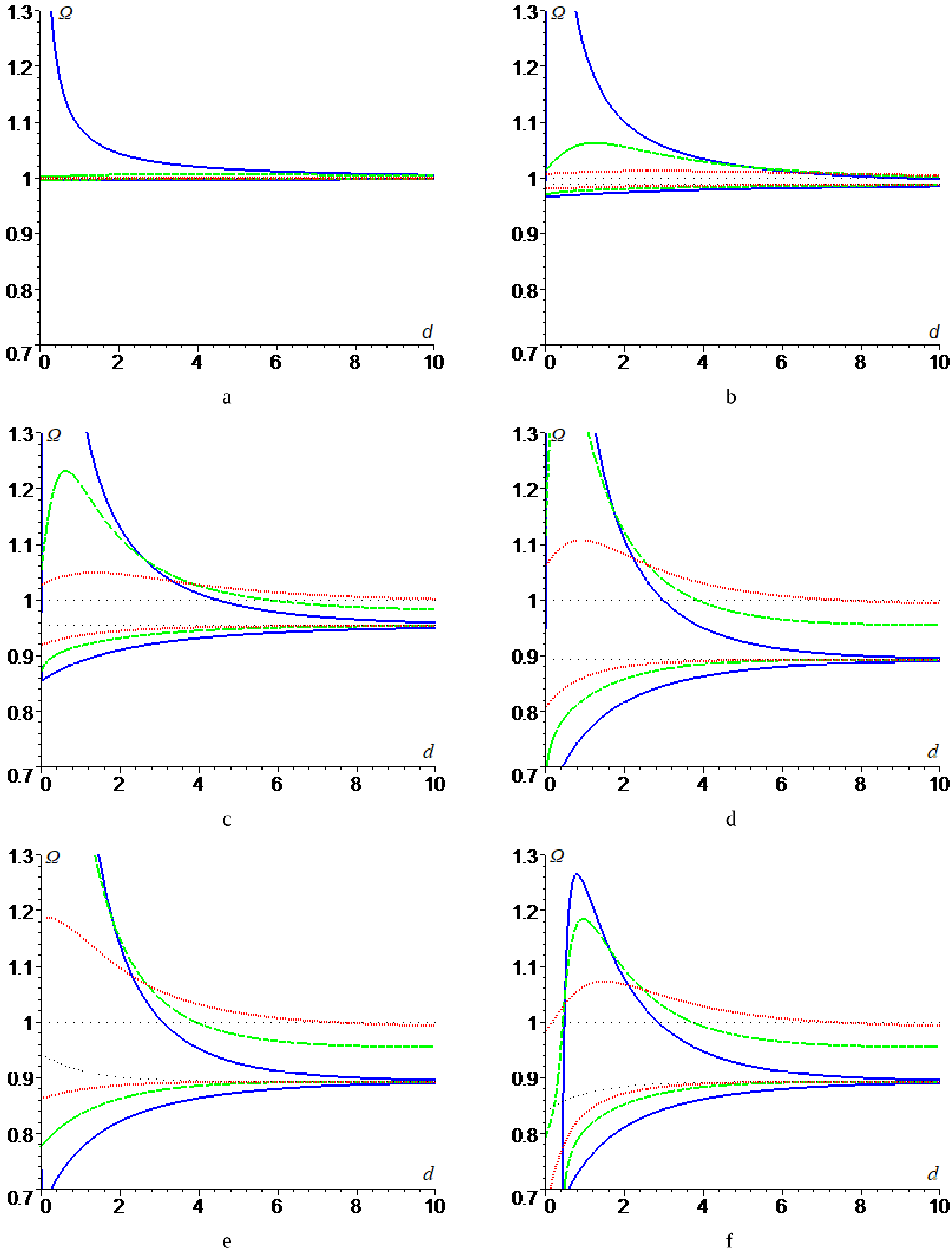

Figure 1. Dependence of frequencies Ω_1 (lower lines) and Ω_2 (upper lines) on the distance between thin layers d with parameters: $\gamma_1 = \gamma_2 = 0$, $h = 0$: a) $\varepsilon_1 = 0.1$, b) $\varepsilon_1 = 0.3$, c) $\varepsilon_1 = 0.6$, d) $\varepsilon_1 = 0.9$; $h = 0$, $\varepsilon_1 = 0.9$: e) $\gamma_1 = \gamma_2 = 1$, f) $\gamma_1 = \gamma_2 = -1$: points – $\varepsilon_2 = 0$, point line – $\varepsilon_2 = \varepsilon_1/3$, dashed line – $\varepsilon_2 = 2\varepsilon_1/3$, solid line – $\varepsilon_2 = \varepsilon_1$.

When $d \to \infty$ we have $e_n = 0$, $E = 0$ and when $\varepsilon_1 > \varepsilon_2$:

$$\Omega_{1,2}^2 = 1 \pm h - \varepsilon_{1,2}^2/4, \qquad \eta_{1,2} = 0. \tag{18}$$

From figure 1a, b, c, d it can be seen that as the distance between the thin layers decreases, frequency Ω_1 decreases the more, the larger the values of ε_1 and ε_2 are. In this case frequency Ω_2 increases much more significantly, but at a certain value of d it begins to decrease. The bigger ε_1 and $\varepsilon_2/\varepsilon_1$, the stronger is the increase in Ω_2 and the smaller is the value of d, after which Ω_2 begins to decrease. Similar is the behaviour of the frequency difference with decreasing d. It increases the more, the greater the value of ε_1 and $\varepsilon_2/\varepsilon_1$, and at a sufficiently small value of d it decreases again. Thus the beat period is reduced to a certain value, and then increases again. For $\gamma_1 = \gamma_2 = 0$ for different values of ε_1 the dependence of the coefficients $\eta_{1,2}$ on the product of $D = \varepsilon_1 d$ and the ratio of $\varepsilon_2/\varepsilon_1$ is almost the same (figure 2). In this case, we assume that $\varepsilon_2 \leq \varepsilon_1$. Coefficient η_1 increases to $D \approx 2$, then decreases, passes through zero and becomes negative. Its value is the bigger (and the value of D, at which the maximum value is reached, and the value of D at which it passes through zero, is the less) the larger the value of $\varepsilon_2/\varepsilon_1$ is. Coefficient η_2 is negative, its modulus also increases to some value of $D < 2$, and then decreases, but does not pass through zero. Thus, as D decreases to a certain value, the amplitude difference at the beats increases. The influence of the LMW existence on a thin layer with a smaller anisotropy inhomogeneity on the LMW amplitude on another thin layer is greater than the influence of the first on the amplitude of the second. With the same anisotropy inhomogeneities and exchange $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $\gamma_1 = \gamma_2 = \gamma$:

$$e_n = \exp(-\varepsilon d/2) = e_d, \qquad E = (1/\varepsilon + d/2)e_d, \tag{19}$$

$k_1 = k_2 = k < 0$ for $\gamma \geq 0$. For $\gamma < 0$ as d decreases less than a certain value, we have $k > 0$. The bigger is $|\gamma|$, the bigger is d at which the sign changes. When $\gamma = -1$ it happens at $d \approx 0,5$. When $k < 0$

$$\Omega_{1,2}^2 = 1 \pm h - \frac{\varepsilon^2}{4} + \frac{[(\gamma\varepsilon/4 - 1)e_d \mp 1]\varepsilon^2 e_d}{2(1 \pm \varepsilon E)}, \qquad \eta_{1,2} = \pm 1. \tag{20}$$

When $d = 0$

$$\Omega_{1,2}^2 = 1 \pm h - 3\varepsilon^2/4 + \gamma\varepsilon^3/16, \qquad \infty, \tag{21}$$

i.e. the frequency Ω_1 decreases to 0.5 with $\varepsilon = 1$, h, $\gamma = 0$, and Ω_2 increases up to infinity. The frequency difference also increases up to infinity, i.e. the periods of oscillations and beats reduce to zero. Thus, for identical thin layers, as the distance decreases, the amplitude difference of the beats does not change, and the period of the beats decreases from infinity to zero. An external magnetic field causes either decrease or increase in the frequencies $\Omega_{1,2}$ and does not affect the coefficients $\eta_{1,2}$ at all. For small d positive γ slightly changes the values of frequencies and coefficients (figure 1e), and negative γ causes more significant changes (figure 1f).

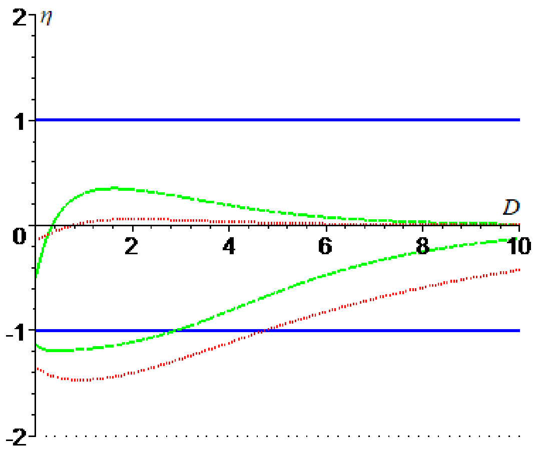

Figure 2. Dependence of the coefficients η_1 (upper lines) and η_2 (lower lines) on the product of the magnetic anisotropy constant inhomogeneity value for the distance between thin layers $D = \varepsilon_1 d$ with parameters $\gamma_1 = \gamma_2 = 0$: points – $\varepsilon_2 = 0$, point line – $\varepsilon_2 = \varepsilon_1/3$, dashed line – $\varepsilon_2 = 2\varepsilon_1/3$, solid line – $\varepsilon_2 = \varepsilon_1$.

3. Conclusion

The dynamics of localized magnetization waves in multilayer ferromagnetic structures was considered. It was shown that in the presence of a five-layer ferromagnetic structure, the equations, describing coupled localized magnetization waves dynamics, are the equations of an oscillatory system with two degrees of freedom. The oscillations represent the sum of two harmonic oscillations and have the form of beats. The dependences of the oscillations frequencies, beats, and coefficients that determine the amplitudes difference during the beats on the distance between thin layers were found. The effect of the external magnetic field, magnetic anisotropy parameters, and exchange interaction in thin layers on these dependences was studied. The values of frequencies and coefficients were calculated for some limiting cases: for an infinite distance between thin layers and for the three-layer structure. In the case of two thin layers with the same magnetic parameters, as the distance between the layers decreases, the beat frequency increases from zero to infinity, and the coefficients are constant and equal to one in absolute value. In the case of two thin layers with different values of magnetic parameters, it was shown that with a decrease in distance between the layers, the beat frequency and the coefficients magnitude first increase from zero to some maximum value, and then decrease. The value of one of the coefficients passes through zero and changes sign.

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